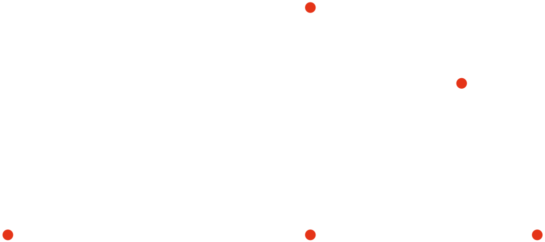
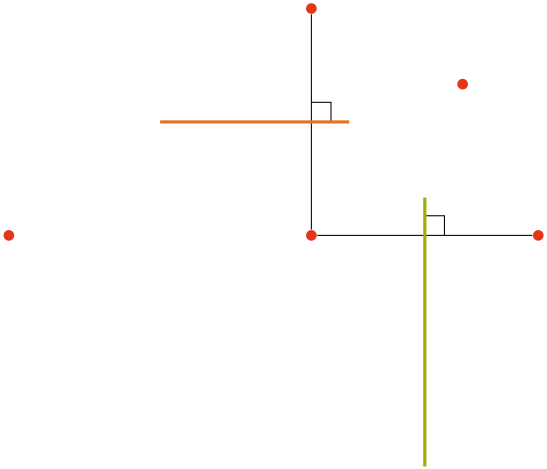
Inzidenz-Strukturen von Powerdiagrammen

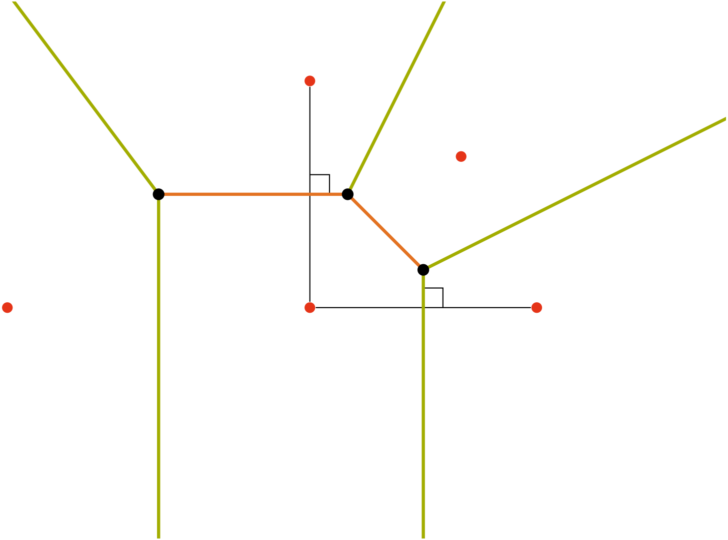
Interdisziplinäres Projekt

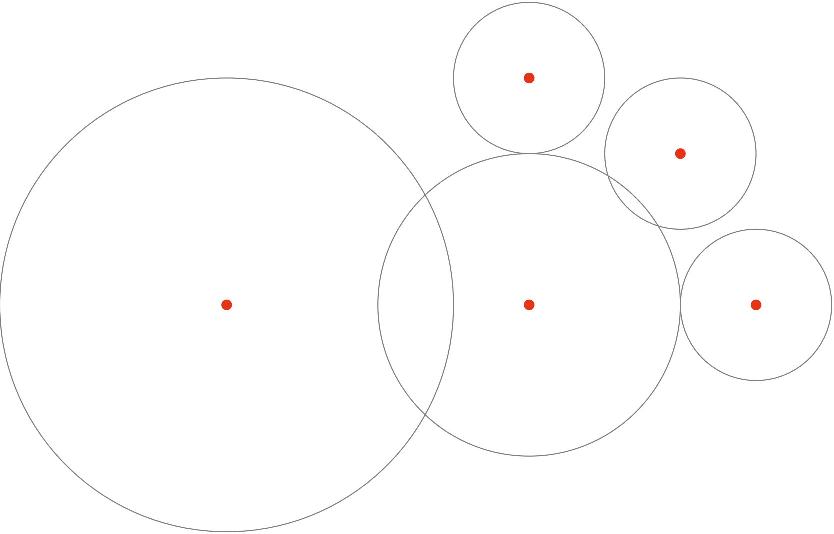
Markus Kaiser

5. Oktober 2015

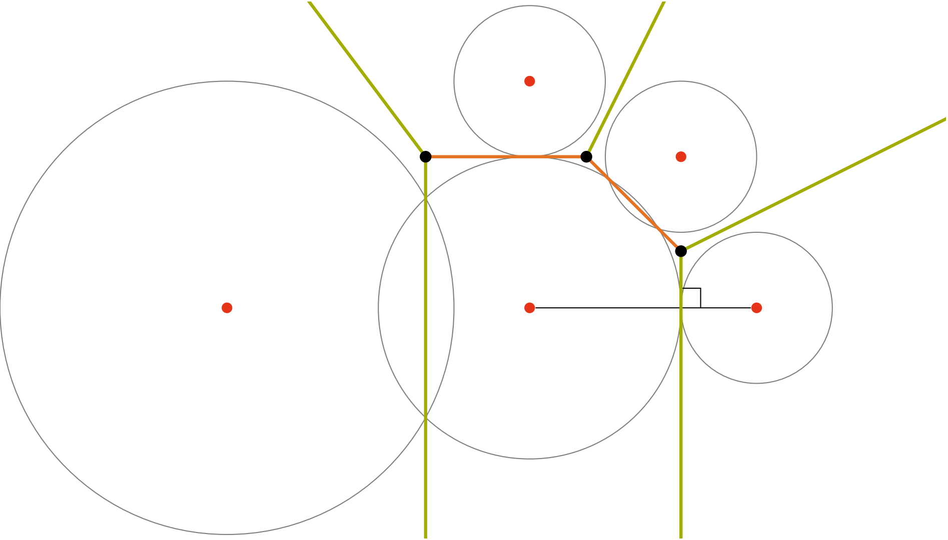








$$\mathrm{pow}(x, s) = \mathrm{d}(x, z)^2 - r^2$$



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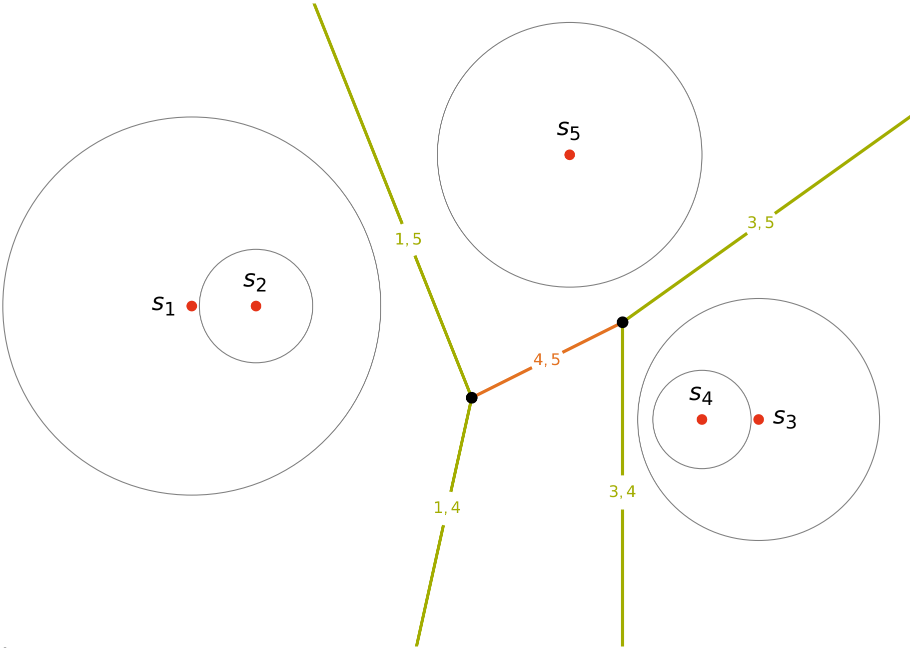
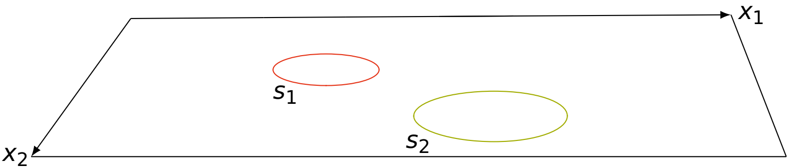
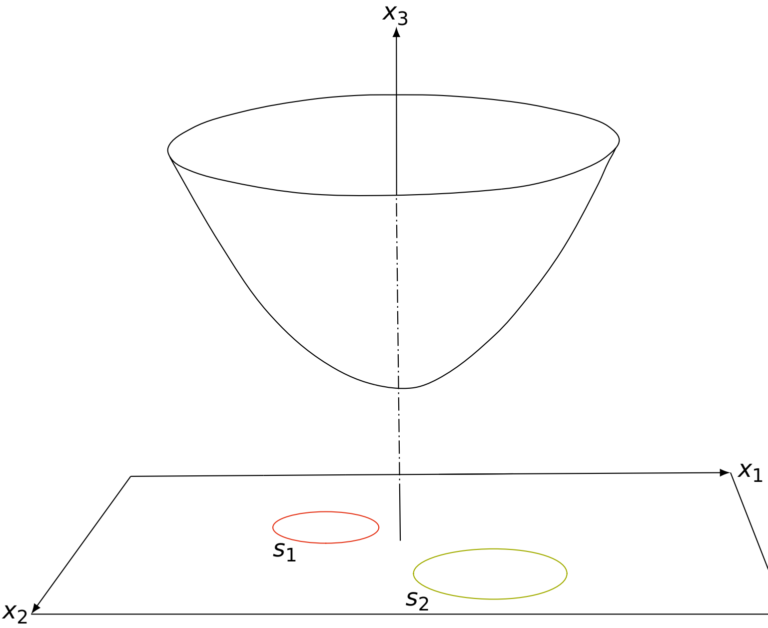
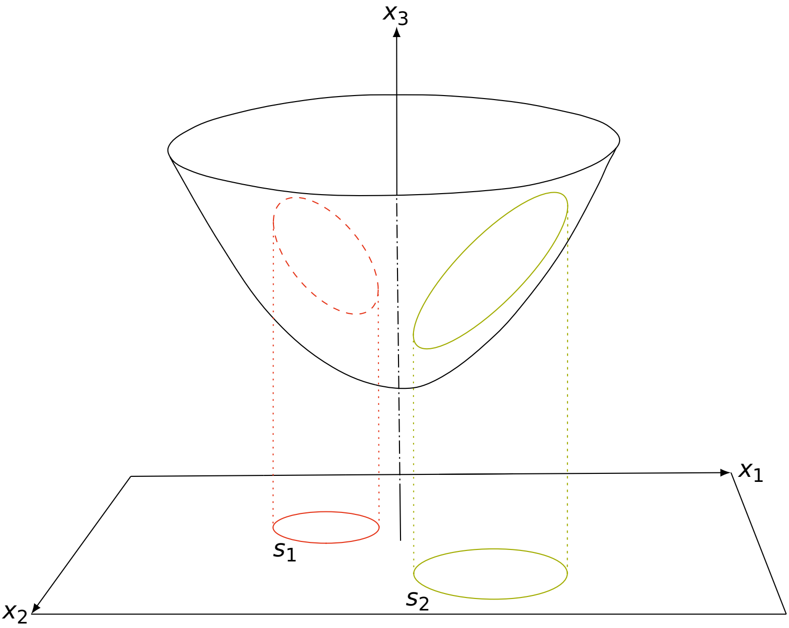

1, 5
s_5
3, 5
s_1
s_2
4, 5
s_4
s_3
1, 4
3, 4


x_1
s_1
s_2
x_2

Transformationsfunktion



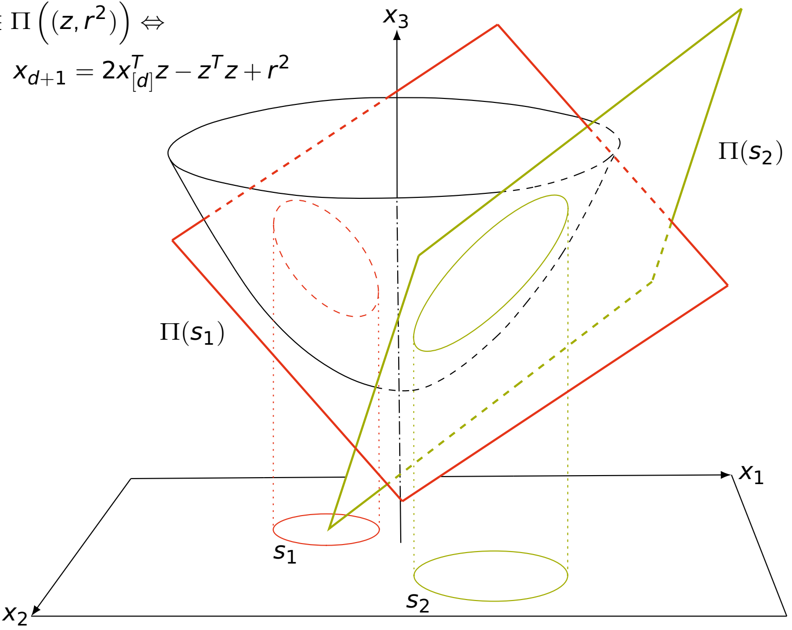


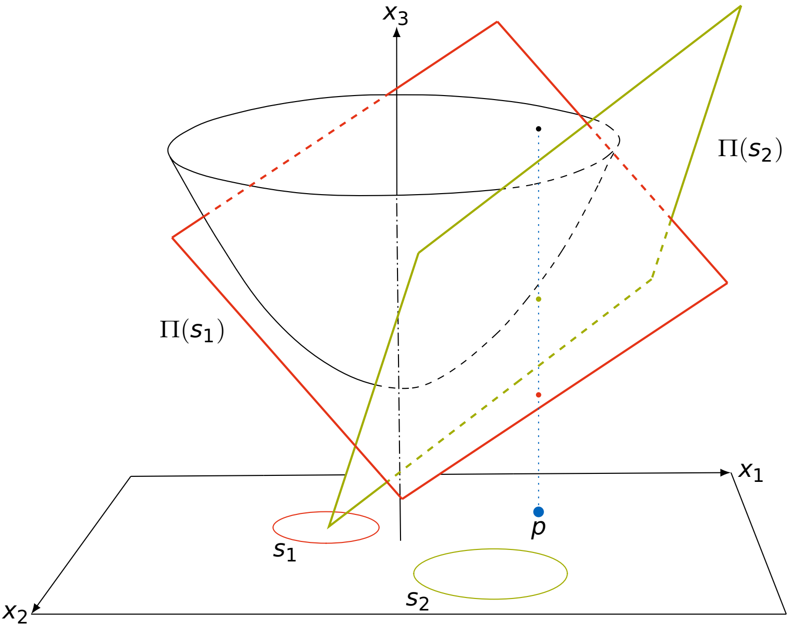
x_3
x_1
x_2
s_1
s_2

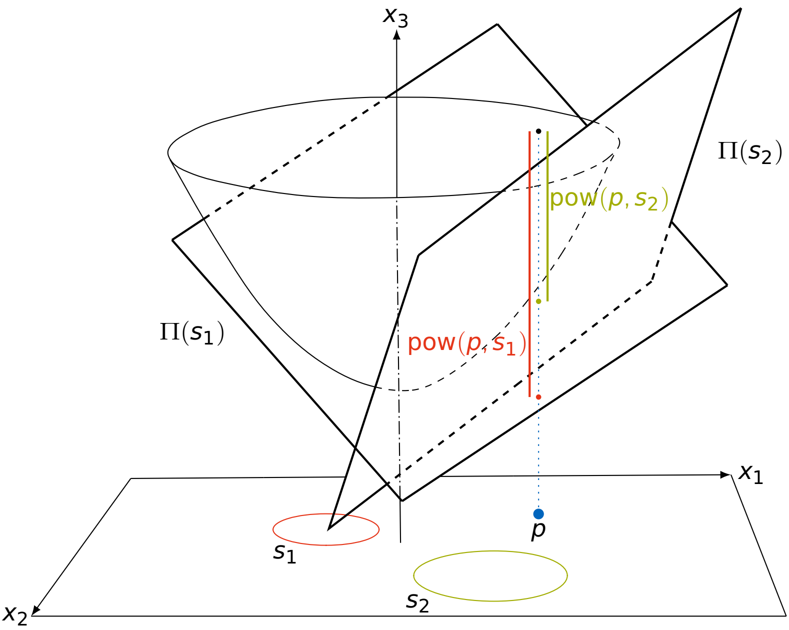
Transformationsfunktion

$$x \in \Pi\left((z, r^2)\right) \Leftrightarrow$$

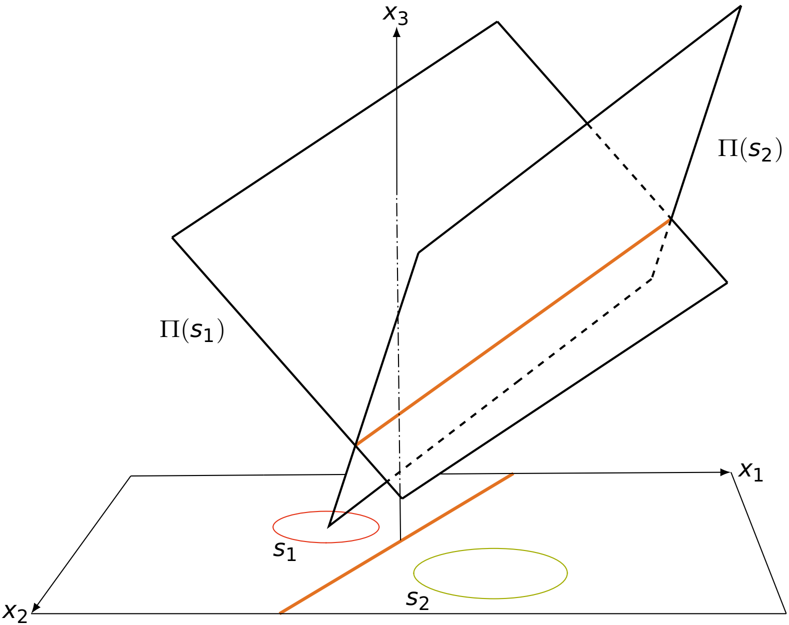
$$x_{d+1} = 2x_{[d]}^T z - z^T z + r^2$$



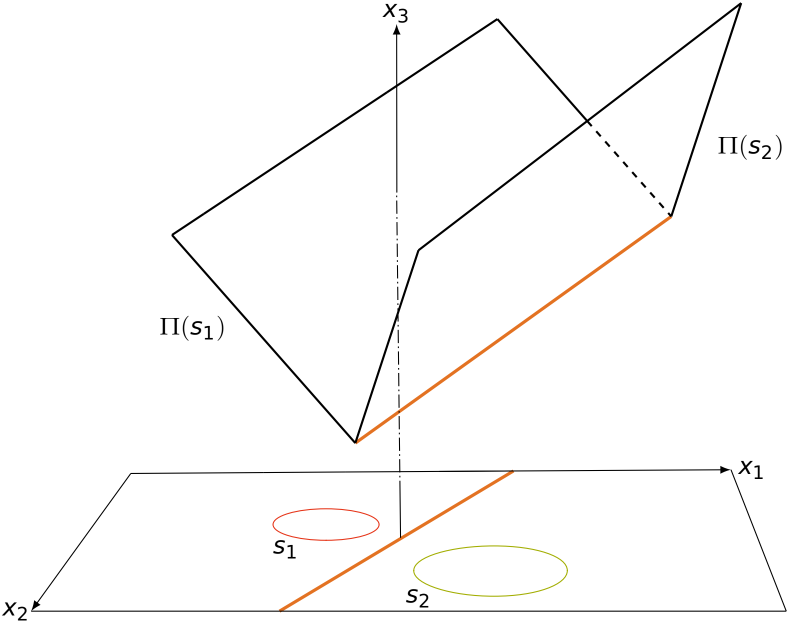

x_3
$\Pi(s_2)$
$\Pi(s_1)$
x_1
p
s_1
s_2
x_2



x_3
$\Pi(s_2)$
$\mathrm{pow}(p, s_2)$
$\Pi(s_1)$
$\mathrm{pow}(p, s_1)$
x_1
p
s_1
s_2
x_2



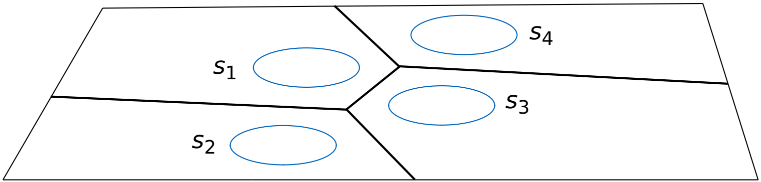
x_3
$\Pi(s_2)$
$\Pi(s_1)$
x_1
s_1
s_2
x_2



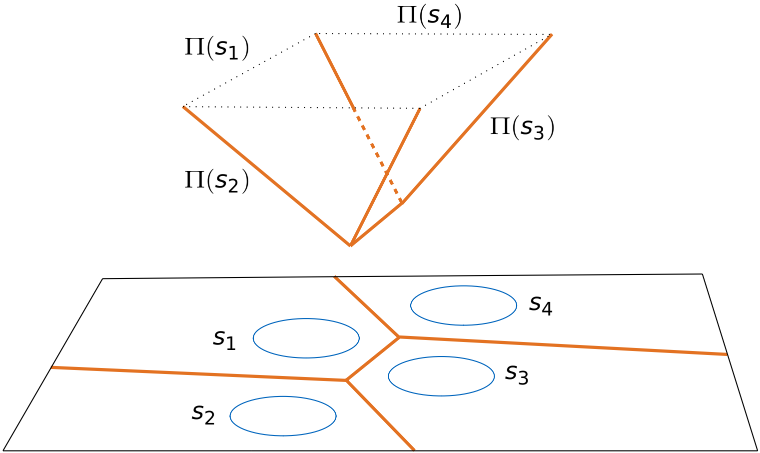
x_3
$\Pi(s_2)$
$\Pi(s_1)$
x_1
s_1
s_2
x_2

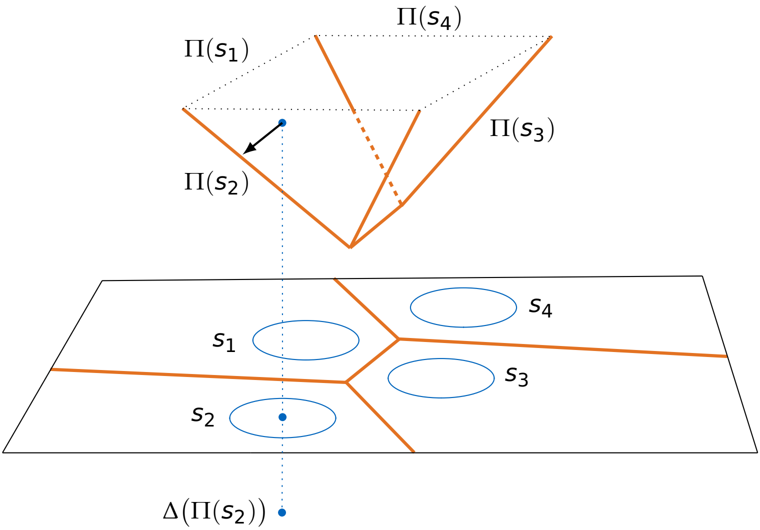
Theorem

Sei P ein $(d+1)$-Polyeder, der nur von Halbräumen beschränkt wird, deren Normalenvektoren nach unten zeigen. Dann existiert ein zu P äquivalentes Powerdiagramm in d Dimensionen und umgekehrt.


s_1
s_2
s_3
s_4

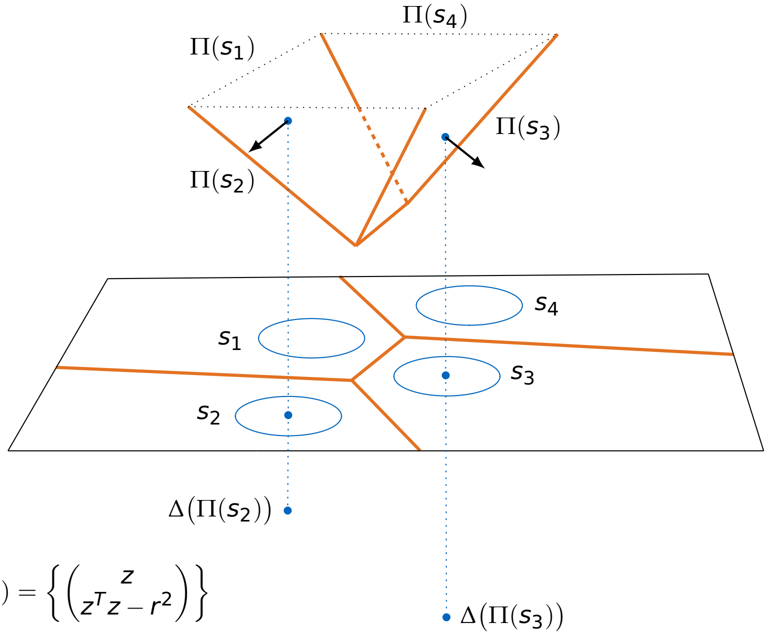
Polarisationsfunktion



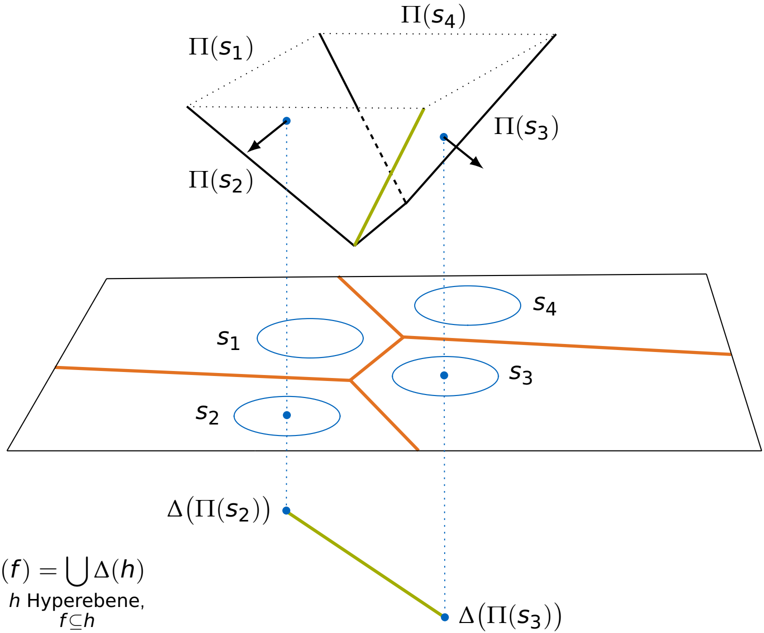


$$\Delta(\Pi(s)) = \left\{ \begin{pmatrix} z \\ z^T z - r^2 \end{pmatrix} \right\}$$

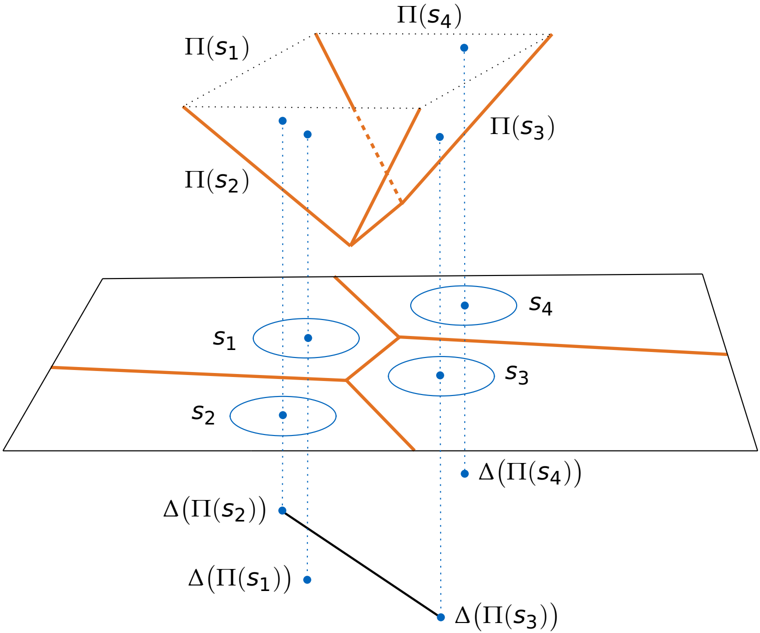
Polarisationsfunktion

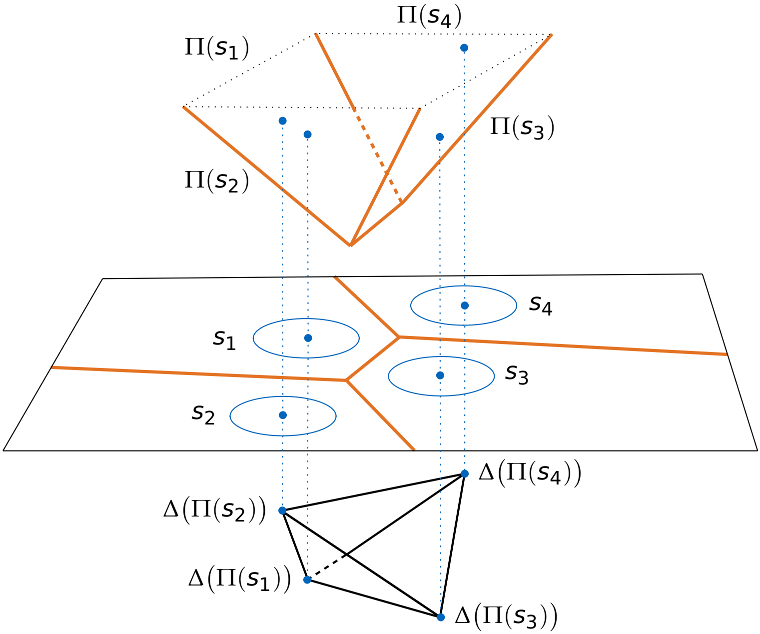


$$\Delta(\Pi(s)) = \left\{ \begin{pmatrix} z \\ z^T z - r^2 \end{pmatrix} \right\}$$



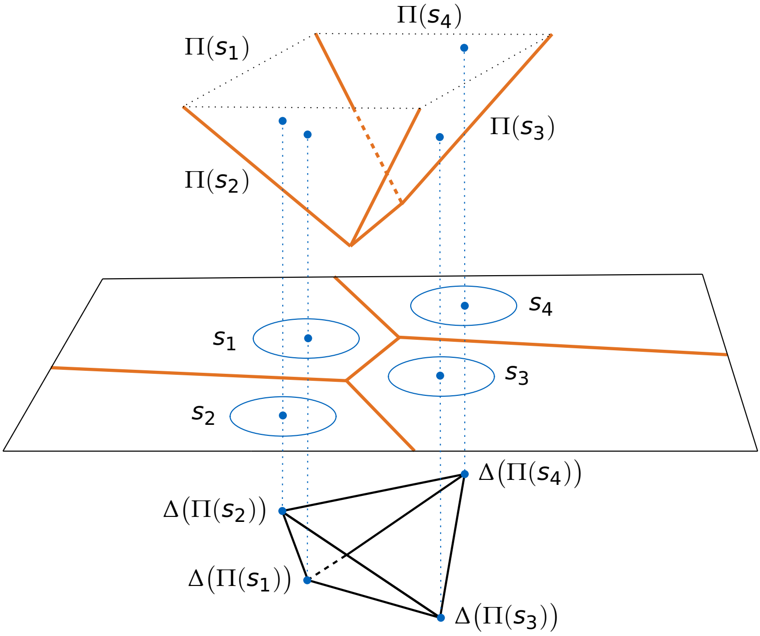
$$\Delta(f) = \bigcup_{\substack{h \text{ Hyperebene,} \\ f \subseteq h}} \Delta(h)$$

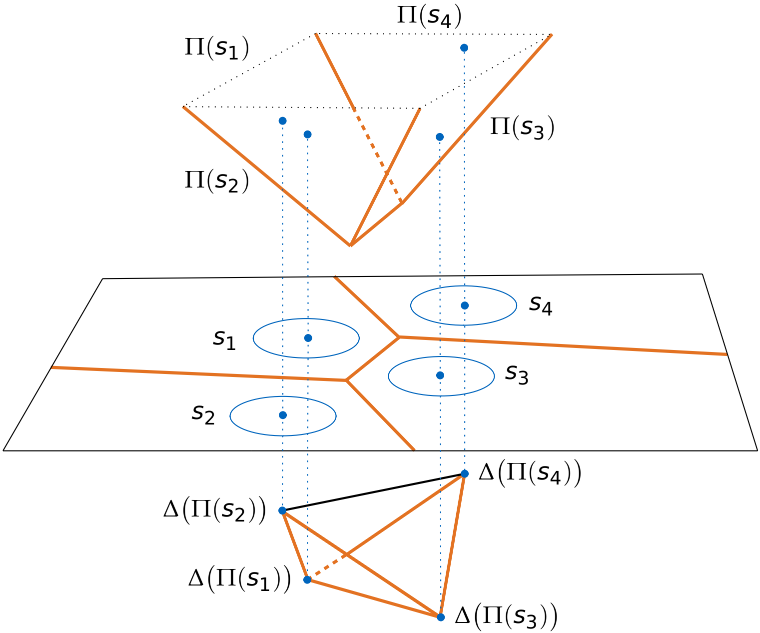

$\Pi(s_4)$
$\Pi(s_1)$
$\Pi(s_3)$
$\Pi(s_2)$
s_4
s_1
s_3
s_2
$\Delta(\Pi(s_4))$
$\Delta(\Pi(s_2))$
$\Delta(\Pi(s_1))$
$\Delta(\Pi(s_3))$


$\Pi(s_4)$
$\Pi(s_1)$
$\Pi(s_3)$
$\Pi(s_2)$
s_4
s_1
s_3
s_2
$\Delta(\Pi(s_4))$
$\Delta(\Pi(s_2))$
$\Delta(\Pi(s_1))$
$\Delta(\Pi(s_3))$

Lemma

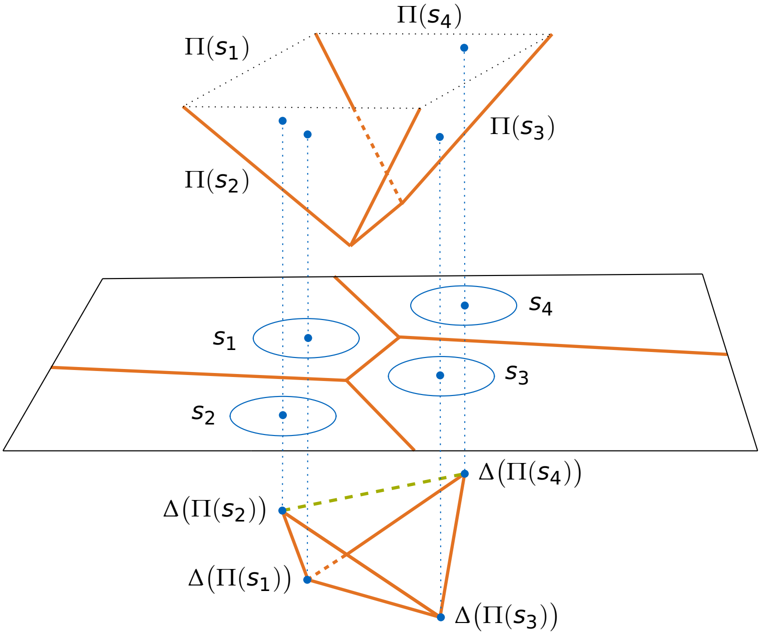
Ein Punkt p ist über, in oder unter einer Hyperebene h genau dann wenn $\Delta(h)$ über, in oder unter $\Delta(p)$ ist.


$\Pi(s_4)$
$\Pi(s_1)$
$\Pi(s_3)$
$\Pi(s_2)$
s_4
s_1
s_3
s_2
$\Delta(\Pi(s_4))$
$\Delta(\Pi(s_2))$
$\Delta(\Pi(s_1))$
$\Delta(\Pi(s_3))$



$\Pi(s_4)$
$\Pi(s_1)$
$\Pi(s_3)$
$\Pi(s_2)$
s_4
s_1
s_3
s_2
$\Delta(\Pi(s_4))$
$\Delta(\Pi(s_2))$
$\Delta(\Pi(s_1))$
$\Delta(\Pi(s_3))$

Polarisationsfunktion

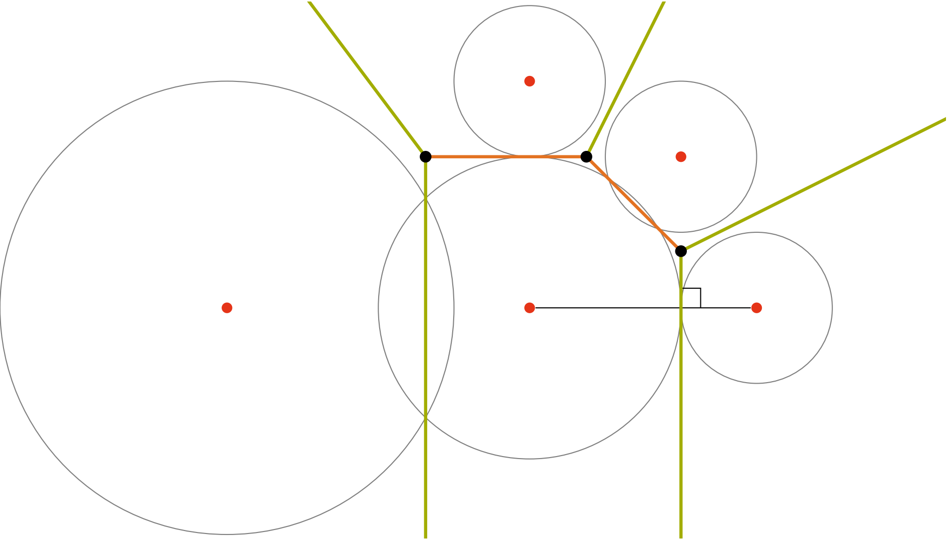


Gegeben eine endliche Menge S von Sphären in d Dimensionen.

Inzidenzstruktur eines Powerdiagrammes bestimmen

1. Bestimme die dualen Punkte $\Delta(\Pi(S))$
2. Berechne die konvexe Hülle dieser Punkte
3. Finde die untere Hälfte der konvexen Hülle
4. Dualisiere diese Hälfte
5. Projiziere die Faces in den Originalraum

- Dominiert durch Konvexe Hülle
- Diese hat Laufzeit $\mathcal{O}(n \log n + n^{\lceil d/2 \rceil})$



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